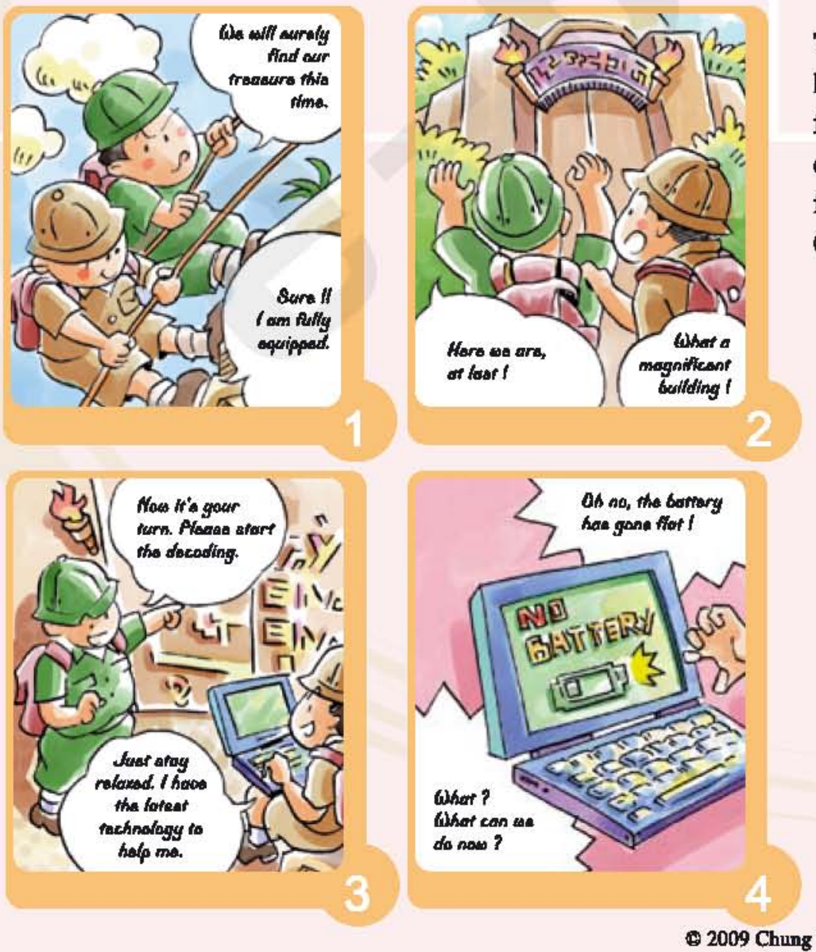
Chapter 2

Basic Algebra

Learning Objectives

After completing this chapter, you will be able to

- appreciate the use of letters to represent numbers.
- rewrite text into algebraic expressions and use text to describe algebraic expressions.
- note the differences between algebraic expressions and arithmetic expressions.
- recognize some common formulae and make use of substitution.
- investigate the patterns of various number sequences, including square sequences, triangular sequences, arithmetic sequences, geometric sequences and Fibonacci sequences.
- use algebraic expressions to represent the nth term of a number sequence.



To get a treasure, two treasure hunters have to complete the following decoding, where each letter represents a number from 0 to 9 without repetition. Can you find the numbers?

$$\begin{array}{rr} & \text{FORTY} \\ & \text{TEN} \\ +) & \text{TEN} \\ \hline & \text{SIXTY} \end{array}$$

Preview

[Basic techniques required for this chapter.]

Basic Technique

1. Addition and subtraction of directed numbers

e.g. **(a)** $(+7)+(+3)=7+3$
$\quad\quad=10$

(b) $(+8)+(-2)=8-2$
$\quad\quad=6$

(c) $(-4)+(+2)=-4+2$
$\quad\quad=-2$

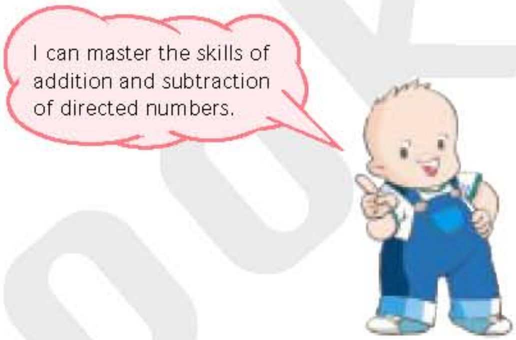
(d) $(-3)+(-6)=-3-6$
$\quad\quad=-9$

(e) $(+7)-(+3)=7-3$
$\quad\quad=4$

(f) $(+8)-(-2)=8+2$
$\quad\quad=10$

(g) $(-4)-(+2)=-4-2$
$\quad\quad=-6$

(h) $(-3)-(-6)=-3+6$
$\quad\quad=3$



2. Multiplication and division of directed numbers

e.g. **(a)** $(+4)(+2)=8$

(b) $(+8)(-4)=-32$

(c) $(-10)(+5)=-50$

(d) $(-12)(-4)=48$

(e) $\dfrac{+4}{+2}=2$

(f) $\dfrac{+8}{-4}=-2$

(g) $\dfrac{-10}{+5}=-2$

(h) $\dfrac{-12}{-4}=3$

2.1 Understanding Algebra

A Introduction

Class Activity 2.1

Aim: To investigate the advantage of using letters to represent numbers

1. A calendar for 2010 from January to March is shown below. Draw at random a 3×3 number square on the calendar each month, and write down the numbers inside the square in the cells provided. (The first one is an example.)

Show Ans.

Jan 2010

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

1: 4	2: 5	3: 6
4: 11	5:	6:
7:	8:	9:

Feb 2010

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28						

1:	2:	3:
4:	5:	6:
7:	8:	9:

Mar 2010

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

1:	2:	3:
4:	5:	6:
7:	8:	9:

2. Observe the numbers in each of the squares, and answer the following questions.

Show Ans.

(a) What is the relation between the numbers in the first cell and the sixth cell?

____

(b) What is the relation between the numbers in the third cell and the seventh cell?

____

3. **(a)** If the number in the first cell is n, express the numbers in the remaining cells in terms of n.

Show Ans.

1 n	2	3
4	5	6
7	8	9

(b) Use the above expressions to represent

(i) the relation obtained in **2(a)**.

____

(ii) the relation obtained in **2(b)**.

____

(c) Can you find out any other relations?

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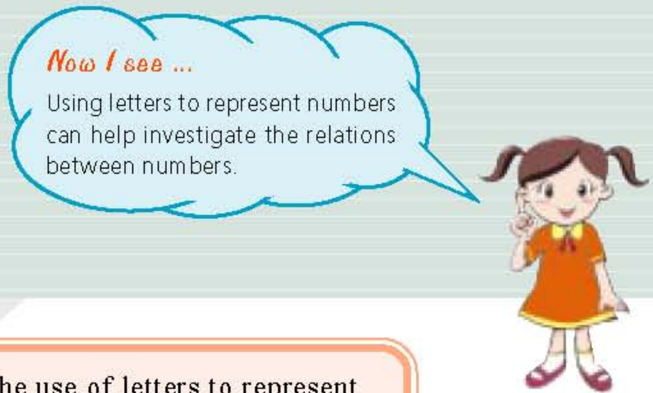


Algebra refers to the use of letters to represent numbers for the investigation of the relations between numbers and the rules of arithmetic.

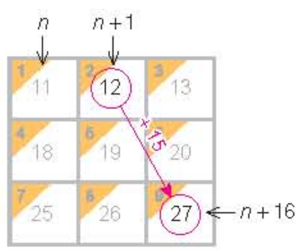
For example, by using the expressions of algebra, we can easily find out the relation between the numbers in the second cell and the ninth cell in the above Class Activity.

$\because$ $(n+1)+15=n+16$

$\therefore$ The number in the second cell plus 15 equals the number in the ninth cell.

◄ '$\because$' means 'because'.

◄ e.g.



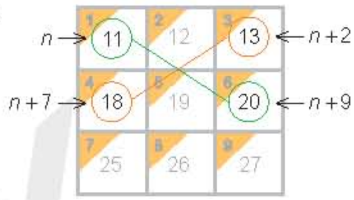
algebra 代數

Besides, do you notice that 'the sum of the numbers in the first cell and the sixth cell is always equal to the sum of the numbers in the third cell and the fourth cell'? Using the rules of arithmetic to be learned in algebra, the following can be easily explained.

Sum of the numbers in the first cell and the sixth cell
$= n+(n+9)$
$= 2n+9$

Sum of the numbers in the third cell and the fourth cell
$= (n+2)+(n+7)$
$= 2n+9$

e.g.



B Algebraic notations and operations

1. Addition and subtraction

> 'a plus b' or 'the sum of a and b' is written as '$a + b$', 'a minus b' is written as '$a - b$'.

(a) $a+b=b+a$
e.g. $3+4=4+3$
(b) $a-b \neq b-a$
('$\neq$' means 'not equal to')
e.g. $4-3 \neq 3-4$

2. Multiplication

> 'a times b' or 'the product of a and b' is written as '$a \times b$', '$a \cdot b$' or 'ab'.

(a) ab is most commonly used.
(b) $a \times b = b \times a$
e.g. $4 \times 5 = 5 \times 4$

When we write algebraic expressions,

(a) the multiplication sign '$\times$' is usually omitted in the following situations:

(i) Multiply a number by a letter
e.g. $6 \times a = 6a$

(ii) Multiply a letter by another letter
e.g. $a \times b = ab$

(b) numbers are usually written first, followed by letters arranged in alphabetical order.

e.g. $a \times 6$ can be written as $6a$.
$x \times 3 \times y$ can be written as $3xy$.

(c) the number 1 is usually omitted when a letter is multiplied by 1.

e.g. $1a$ can be written as a.

3. Division

> 'Dividing a by b' is written as '$a \div b$' or '$\frac{a}{b}$'.

(a) $\frac{a}{b}$ is most commonly used.
(b) $a \div b \neq b \div a$
e.g. $4 \div 2 \neq 2 \div 4$

For algebraic operations, multiplication and division should be done first, and then followed by addition and subtraction. Meanwhile, the use of brackets should also be paid attention to.

e.g. $a \div (2+b) = \frac{a}{2+b}$

$a \div 2 + b = \frac{a}{2} + b$

$a \div (2 \times b) = a \div 2b = \frac{a}{2b}$

$a \div 2 \times b = \frac{a}{2} \times b = \frac{ab}{2}$

$a \div 2b$ means $a \div (2b)$

4. Index notation

Index notation can be used to express the multiplication of the same letters.

e.g. $a \times a \times a \times a = a^4$

$xyy = xy^2$

$5 \times 5 \times a \times a \times c \times c \times c = 5^2 a^2 c^3$

(a) a^4 is read as a to the power 4.
(b) $a \times a \times a \times a$ is the expanded form of a^4.
e.g. $3 \times 3 \times 3 \times 3 = 3^4$

5. Algebraic expression

> **Algebraic expressions** are expressions obtained by addition, subtraction, multiplication or division of numbers and letters representing numbers.

e.g. $3x + 1$, $2x - 5$, $10xy$, $\frac{6x - y}{4}$

Additional Example 2.1

Example 2.1 Using algebraic expressions to represent statements

Express the following statements by using algebraic expressions.

(a) Add y to x, and then multiply the sum by z.

(b) Subtract a from b, and then divide the difference by c.

Classwork 2.1

Solution

Express the following statements by using algebraic expressions.

(a) Subtract 4 from z.
(b) Multiply 3 by y.
(c) Add x to 2, and then multiply the sum by m.
(d) Divide the sum of a and b by c.

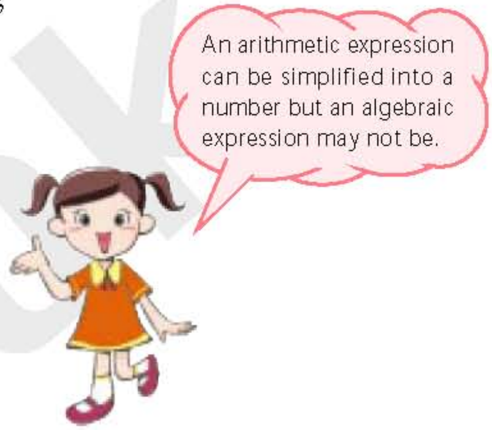
algebraic expression 代數式

Solution

(a) Adding y to x in algebraic expression is $x+y$.
Multiplying the sum by z becomes $\underline{\underline{(x+y)z}}$.

(b) Subtracting a from b in algebraic expression is $b-a$.
Dividing the difference by c becomes $\underline{\underline{\dfrac{b-a}{c}}}$.

The main difference between arithmetic expressions and algebraic expressions is that algebraic expressions involve letters representing numbers while arithmetic expressions do not.



	Arithmetic expression	Algebraic expression
Addition	'5 plus 4' can be written as '5 + 4'	'a plus b' can be written as '$a+b$'
Subtraction	'8 minus 3' can be written as '8 – 3'	'a minus b' can be written as '$a-b$'
Multiplication	'7 times 2' can be written as '7×2'	'a times b' can be written as '$a\times b$', '$a\cdot b$' or 'ab'
Division	'Divide 10 by 3' can be written as '$10\div 3$' or '$\dfrac{10}{3}$'	'Divide a by b' can be written as '$a\div b$' or '$\dfrac{a}{b}$'

Table 2.1

Additional Example 2.2

Example 2.2 Simplification by using multiplication or division

Simplify the following expressions.

(a) $a+a+a+a$ **(b)** $6-2\times b$
(c) $3c\times 4$ **(d)** $d\div 5\times 20$

Solution

(a) $a+a+a+a=4\times a$
$=\underline{\underline{4a}}$

(b) $6-2\times b=\underline{\underline{6-2b}}$

(c) $3c\times 4=4\times 3\times c$
$=12\times c$
$=\underline{\underline{12c}}$

(d) $d\div 5\times 20=\dfrac{d}{5}\times 20$
$=d\times 4$
$=\underline{\underline{4d}}$

Classwork 2.2

Solution

Simplify the following expressions.
(a) $p+p+p+p-p$
(b) $2+q\times 7$
(c) $r\times 5\times 3$
(d) $s\times 24\div 6$

Additional Example 2.3

Example 2.3 Simplification by using addition, subtraction or index notation

Simplify the following expressions.

(a) $2y+3y-y$

(b) $pppp$

(c) $3\times qq\times 2\times q\times rr$

(d) $7+3x-2+4x$

Solution

(a)
$$\begin{aligned}2y+3y-y &= (y+y)+(y+y+y)-y\\ &= y+y+y+y+y-y\\ &= \underline{\underline{4y}}\end{aligned}$$

(b)
$$\begin{aligned}pppp &= p\times p\times p\times p\\ &= \underline{\underline{p^4}}\end{aligned}$$

(c)
$$\begin{aligned}3\times qq\times 2\times q\times rr &= 3\times 2\times q\times q\times q\times r\times r\\ &= 6\times q^3\times r^2\\ &= \underline{\underline{6q^3r^2}}\end{aligned}$$

(d)
$$\begin{aligned}7+3x-2+4x &= 7+3x+(-2)+4x\\ &= 7+(-2)+3x+4x\\ &= 7-2+3x+4x\\ &= \underline{\underline{5+7x}}\end{aligned}$$

Classwork 2.3 Solution

Simplify the following expressions.

(a) $4a-2a+5a$

(b) $bb\times bbb$

(c) $c\times cc\times 7\times 2c$

(d) $2d+5+4d-9$

Additional Example 2.4

Example 2.4 Using algebraic expressions to represent quantities

Winnie is now 4 times as old as Joe. If Winnie's present age is x, answer the following questions by using algebraic expressions.

(a) Find the present age of Joe.

(b) Find Winnie's age three years ago.

Solution

(a) The present age of Joe is $\underline{\underline{\dfrac{x}{4}}}$.

(b) Three years ago, Winnie's age was $\underline{\underline{x-3}}$.

Classwork 2.4 Solution

In a Mathematics test, Andy's score is 3 times that of Jimmy's. If Jimmy's score is x, answer the following questions by using algebraic expressions.

(a) Find Andy's score.

(b) If Ben's score is higher than Andy's by 10, find Ben's score.

Additional Example 2.5

Example 2.5 Application of algebraic operations

In a supermarket, the prices of an orange and an apple are $\$r$ and $\$s$ respectively.

(a) Susan gets 5 oranges and 4 apples. How much are they altogether?

(b) At the cashier, Susan takes away 2 oranges but gets an extra box of orange juice which costs \$13. How much does Susan have to pay?

(c) Susan pays the bill by an Octopus card. If \$80 is left in the Octopus card after the payment, what is the stored value of the Octopus card originally?

Solution

(a) Total cost of the items

$= 5 \times \$r + 4 \times \$s$

$= \$(5 \times r + 4 \times s)$

$= \underline{\underline{\$(5r + 4s)}}$

(b) Total cost of the items

$= \$(5r + 4s) - \$2r + \$13$

$= \$(5r + 4s - 2r + 13)$

$= \$(5r - 2r + 4s + 13)$

$= \underline{\underline{\$(3r + 4s + 13)}}$

(c) The stored value of the Octopus card originally

$= \$(3r + 4s + 13) + \$80$

$= \$(3r + 4s + 13 + 80)$

$= \underline{\underline{\$(3r + 4s + 93)}}$

Classwork 2.5

Solution

The daily *expenditure* of Stephen from Monday to Friday is $\$a$, and that on Saturday and Sunday is $\$b$.

(a) Find the expenditure of Stephen each week.

(b) A calendar for September is shown below. Find the expenditure of Stephen this month.

SEP

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

(c) Due to the rainstorm, Stephen stayed at home on the 14th and 15th of September without spending any money. Find the actual expenditure of Stephen this month.

Skills Upgrading Corner 2.1

Solution

1. Express the following algebraic expressions in the expanded form.

   (a) ab^3 **(b)** $5x^2y$

   (c) $2b^2d^2$ **(d)** $3p^3q^2$

Solution

2. Simplify the following expressions.

   (a) $7a + 2b - 9a - 8b + 6b$

   (b) $n \times 5m \times 4n \times m \times m$

expenditure 開支

Solution

3. Is 'the simplification of $a+a$ is a^2' correct ? Explain your answer.

Solution

4. A girl has \$$x$, and her brother has \$20 more than she has. Answer the following questions by using algebraic expressions.
   (a) How much does her brother have?
   (b) How much do they have altogether?

Exercise 2A

Level 1

1. Express the following statements by using algebraic expressions.
   (a) x plus 12.
   (b) Multiply x^2 by y.
   (c) 5 is subtracted from 3 times y.
   (d) $2x$ is subtracted from the product of x and y.
   (e) The sum of p and q is multiplied by 4.
   (f) Divide the sum of x and y by $2z$.
   (g) $3x$ minus 5, and then multiply the difference by y.
   (h) 2 times x is divided by 5 times y^2.
   (i) Divide the sum of 3 times a and 4 times b by the sum of a and b.
   (j) 2 times the square of a is added to 3 times the square of b.

Simplify the following expressions. **(2 – 3)**

2. **(a)** $a+b+b+a+b$
   (b) $3+5\times e$
   (c) $f\div 9\times 27$
   (d) $(5\times g)\div(h\times k)$
   (e) $p\times 2q\times 3p\times q$
   (f) $4r\times r\times 2s\times r\times s$
   (g) $(2\times \ell-3\times m)\div(5\times n\times n)$
   (h) $2\times 2c+3\times 2d\div(18\times b\times b)$

3. **(a)** $2r+6r+3r$
   (b) $3m+4m-6m$
   (c) $8x-x+4+6$
   (d) $3y+9-y+11$
   (e) $2x+5y+4x-3y$
   (f) $6a+3b-2b+a$
   (g) $5b-2d+b+4d-3b$
   (h) $10h+7k-4k-8h+6k$

Use the simplest algebraic expressions to answer the following questions. **(4 – 7)**

4. Jason is now x years old and his father is 4 times as old as he is. Find the age of his father.

5. Mark is now m years old. Two years ago, his father was 5 times as old as he was. What was his father's age two years ago?

6. The smallest number of three *consecutive numbers* is w. Find their sum.

7. The smallest number of three consecutive odd numbers is $2n+1$. Find their sum.

Level 2

8. The weight of May is $\frac{1}{5}$ of her mother's.

(a) If May's weight is m kg, find her mother's weight.

(b) Find the sum of their weights.

9. Jacky has n books. The number of books owned by Jeffrey is 3 more than two times the number of books owned by Jacky.

(a) How many books does Jeffrey have?

(b) How many books does Jeffrey have more than Jacky?

10. (a) A bag of marbles is evenly *distributed* to 8 children. After each of them gets p marbles, 7 marbles are left. How many marbles are there in the bag originally?

(b) If the marbles can be evenly distributed to 9 children, how many marbles does each child get in this case?

11. Mabel rides a bike at a speed of v km/h.

(a) Find the time taken for her to travel 10 km.

(b) If she increases her speed by 3 km/h, find the time taken for her to travel 10 km.

[Hint: Distance = Speed × Time]

12. Mr. Lee is now x years old, and he is 4 years older than his wife.

(a) Find the present age of Mrs. Lee.

(b) After 10 years, what is the sum of their ages?

(c) After 10 years, the sum of their ages will be equal to 5 times the age of their daughter. What is the present age of their daughter?

13. Use a real-life situation to represent each of the following algebraic expressions.

(a) $a+10$ **(b)** $3b$ **(c)** $x+2y$

consecutive numbers 連續數 distribute 分配

2.2 Formulae and Substitution

Class Activity 2.2

Aim: To learn how to set up formulae and make use of substitution

The Body Mass Index (BMI), introduced by the World Health Organization, is the most commonly used index to measure the body figure. The computation method is shown below.

BMI is equal to the weight (kg) divided by the square of height (m^2).

According to the organization's suggestion, the ideal BMI for Asian is between 18.5 and 22.9.

Health status	 Extremely fat	 Overweight	 Normal weight	 Underweight
BMI	Greater than 24.9	Between 23.0 and 24.9	Between 18.5 and 22.9	Less than 18.5

* BMI formula is not suitable for people under 18, *bodybuilding athletes*, *pregnant* or *breastfeeding* women, and elderly with long-term health conditions.

(a) Let BMI be b, the height be h m and the weight be w kg. Express the BMI formula in terms of b, h and w. Show Ans.

____

(b) According to the above formula, assess the health status of the following three adults. Show Ans.

Don	Joey	Leo
		
Height: 150 cm	Height: 160 cm	Height: 180 cm
Weight: 63 kg	Weight: 51.2 kg	Weight: 48.6 kg
w = ________	w = ________	w = ________
h = ________	h = ________	h = ________
b = ________	b = ________	b = ________
Health status of Don: ________	Health status of Joey: ________	Health status of Leo: ________

bodybuilding athlete 從事肌肉訓練的運動員　pregnant 懷孕　breastfeeding 授乳

(c) For an adult with a height of 170 cm, what is the range of his weight to be regarded as normal?

Show Ans.

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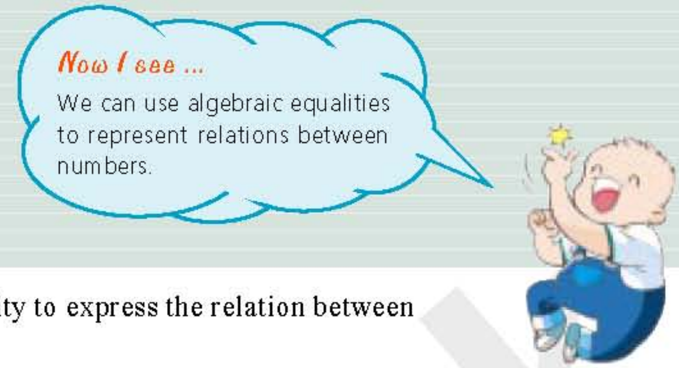


In Class Activity 2.2, we use an equality to express the relation between b, w and h as follows.

$$b = \frac{w}{h^2}$$

This kind of expression is called a **formula**.

> In a formula, each letter represents a number, and the value of the number can vary.

e.g. In Class Activity 2.2, w represents weight, and the value of w can be 63, 51.2 or 48.6.

> In a formula, the value of a letter can be found if we know the values of all other letters.

e.g. When $w = 63$ and $h = 1.5$, $b = \dfrac{63}{1.5^2}$.

This method of replacing letters with values of the numbers is called **substitution**.

Additional Example 2.6

Example 2.6 Finding the values by substitution

If $a = 3$ and $b = 8$, find the values of the following expressions.

(a) ab **(b)** $3a + 5b$

(c) $5a(a + b)$ **(d)** $(a + b)^2$

Solution

(a) $ab = (3)(8)$
$= \underline{\underline{24}}$

◄ $ab = a \times b$
$= 3 \times 8$

Classwork 2.6

Solution

If $x = 4$ and $y = 2$, find the values of the following expressions.

(a) $x(4 - y)$ (b) $\dfrac{x + 4}{y + 2}$

(c) $x^2 - y^2$ (d) $(x - y)^2$

formula 公式 substitution 代入法

(b) $3a+5b=3(3)+5(8)$

$=9+40$

$=\underline{\underline{49}}$

(c) $5a(a+b)=5(3)(3+8)$

$=5(3)(11)$

$=\underline{\underline{165}}$

(d) $(a+b)^2=(3+8)^2$

$=11^2$

$=\underline{\underline{121}}$

Example 2.7 Setting up formulae

Additional Example 2.7

The monthly income $\$w$ of a salesman is the sum of the basic salary and bonus. Suppose the basic salary is \$3 000 and a bonus of \$200 will be paid for selling each product, write down the formula for the monthly income of a salesman selling n products a month.

Solution

Monthly income = Basic salary + Bonus

$\$w=\$3\ 000+\$200\times n$

$=\$(3\ 000+200n)$

$\therefore$ $\underline{\underline{\text{The formula for the monthly income is } w=3\ 000+200n.}}$

Classwork 2.7

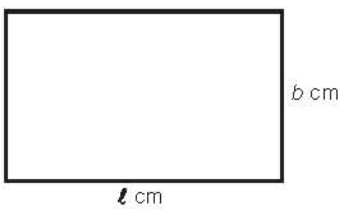
Solution

The cost $\$C$ of publishing each issue of magazine is the sum of the printing cost \$4 000 and the cost of paper. If the cost of paper of one copy of magazine is \$3, write down the cost formula for publishing y copies of magazine.

Example 2.8 Application of substitution

Additional Example 2.8

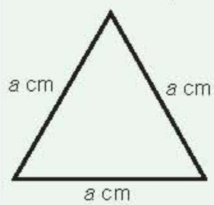
The formula for the perimeter P cm of a rectangle is $P=2(\ell+b)$, where ℓ cm and b cm are the length and breadth respectively. If the length and breadth of a rectangle are 7 cm and 5 cm respectively, find the perimeter of the rectangle.



Classwork 2.8

Solution

The formula for the perimeter P cm of an equilateral triangle is $P=3a$, where a cm is the length of a side.



(a) If $a=4$, find the value of P.
(b) If $a=9$, find the value of P.

Solution

Given $\ell = 7$ and $b = 5$,

$$\begin{aligned} P &= 2(\ell + b) \\ &= 2(7+5) \\ &= 2(12) \\ &= 24 \end{aligned}$$

$\therefore$ <u>The perimeter of the rectangle is 24 cm.</u>

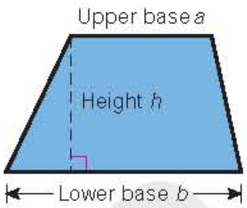
Skills Upgrading Corner 2.2

Solution

1. In primary school, we have learned how to evaluate the area of a trapezium:

‘The area of a trapezium can be obtained by multiplying the sum of the upper base and the lower base by the height and then dividing the product by 2.’

(a) Let the upper base be a, the lower base be b, the height be h and the area be A. Write down the formula for the area of a trapezium.



(b) Use the formula in **(a)** to complete the following table.

a	b	h	Formula ( )
2	5	8	
3	9	5	
4	8	3	

Solution

2. If $a = 5$, $b = 3$ and $c = 2$, find the values of the following expressions.

(a) $a + 2b - c$ **(b)** $a^2b - 9c$

(c) $(a + b) \div 2c$ **(d)** $2a \div (b + c)$

(e) $\dfrac{ab}{5} + \dfrac{a^2 + b^2}{c}$ **(f)** $(a + b^2 - 5c)^2$

Exercise 2B

Level 1

1. If $x=4$, find the values of the following expressions.

(a) $x+8$ **(b)** $5x$ **(c)** $24 \div x$

(d) $30-5x$ **(e)** $3(x+4)$ **(f)** x^2-5

2. If $y=-2$, find the values of the following expressions.

(a) $6-y$ **(b)** $4y^2$ **(c)** $-32 \div y$

(d) $(3+y)^2$ **(e)** $(y-1)(y+2)$ **(f)** $\dfrac{y+12}{y}$

3. Find the value of each of the following expressions by substituting the values given in the brackets [ ] into the expression.

(a) $3(x+y)-7$ $[x=2,\ y=3]$

(b) $(4x+3+2y)(2y-3+4x)$ $[x=\dfrac{1}{2},\ y=4]$

(c) $\dfrac{1}{x^2+2y^2}$ $[x=2,\ y=3]$

4. The following table shows the values represented by a to j.

Letter	a	b	c	d	e	f	g	h	i	j
Value	1	2	3	4	5	6	7	8	9	0

Find the values of the following expressions.

(a) $ac+bd$ **(b)** $\dfrac{e+f}{h}$ **(c)** $i(d+2c)$

(d) $(d-b)(f+c)$ **(e)** ah^2g **(f)** $g^2+h^2+j^2$

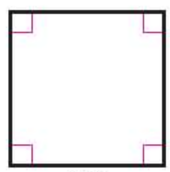
(g) $2(3i-b)\div 5b$ **(h)** $f(5h^2-2j)$ **(i)** $(a+b+c+d+e+f)\div ghi$

5. In each of the following expressions, determine whether it is an algebraic expression or a formula.

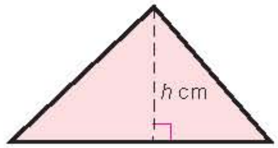
(a) $y=2x$ **(b)** $6ab^2$ **(c)** $A=\ell h$

(d) $E=mc^2$ **(e)** $\pi r^2 h$ **(f)** $6-3r^2$

6. The length of each side of a square is x cm. If the perimeter of the square is S cm, write down the formula for the perimeter of the square.



7. The base of a triangle is twice its height of h cm. If the area of the triangle is A cm^2, write down the formula for the area of the triangle.

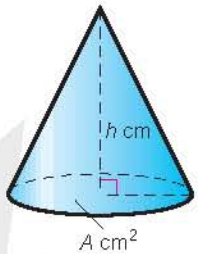


Level 2

8. The figure shows a cone with a height of h cm and a base area of A cm^2. It is known that the volume V cm^3 of the cone is given by the formula $V = \dfrac{Ah}{3}$.

(a) If $A = 10$ and $h = 6$, find the value of V.

(b) If $A = 1$ and $h = 7$, find the value of V.



9. If 1, 2, 3, $\cdots$, n are n consecutive numbers and their sum S(i.e. $1+2+3+\cdots+n$) is given by $S = \dfrac{n(n+1)}{2}$, find the value of S in each of the following cases.

(a) $n = 100$

(b) $n = 500$

(c) $n = 800$

10. A wooden bridge can support a weight up to 500 kg. There are now x adults and y children, with a total weight of W kg, passing the bridge at the same time. Suppose each adult weighs 60 kg and each child weighs 30 kg,

(a) write down a formula about W.

(b) If there are 7 adults and 2 children passing the bridge at the same time, can the bridge support their total weight? Explain your answer.

11. '*Braking distance*' refers to the distance d m travelled by a vehicle before coming to a stop after applying the brake.

After several *trials*, it is found that braking distance can be obtained by the following formula:

$$d = \frac{v^2}{20k}$$

where v m/s is the speed of the vehicle when the brake is just applied and k is a specific value.

braking distance 煞車距離 trial 試驗

(a) Express the speed of 43.2 km/h of a vehicle in m/s.

(b) When the weather is good, k is 0.6. Complete the following table.

Speed (km/h)	Braking distance (m)
43.2	
54	
97.2	

(c) A driver is driving at a speed of 54 km/h and he applies the brake immediately after he notices an object in front of him. The distance between the vehicle and the object at that moment is 20 m.

(i) If he is driving in a good weather condition, determine whether the vehicle will crash into the object.

(ii) If he is driving in a rainy day, where k is 0.55, determine whether the vehicle will crash into the object.

2.3 Sequences

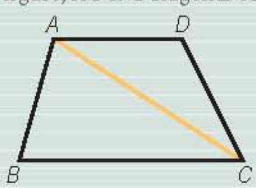
A Introduction

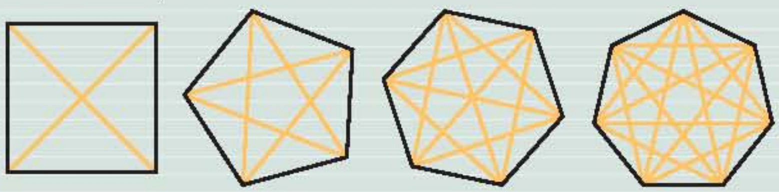
Class Activity 2.3

Aim: To investigate the patterns of sequences

1. How many diagonals does each of the following polygons have? Write down your answers in the table below.

A diagonal of a polygon is a line segment joining two non-adjacent vertices of the polygon. e.g. In the figure, AC is a diagonal of $ABCD$.



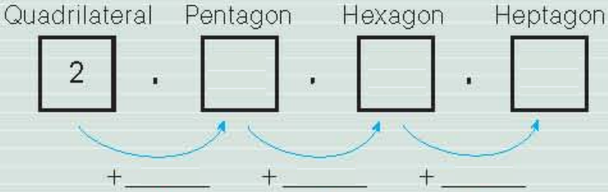


Quadrilateral Pentagon Hexagon Heptagon

Polygon	Number of diagonals
Quadrilateral	2
Pentagon	
Hexagon	
Heptagon	

Show Ans.

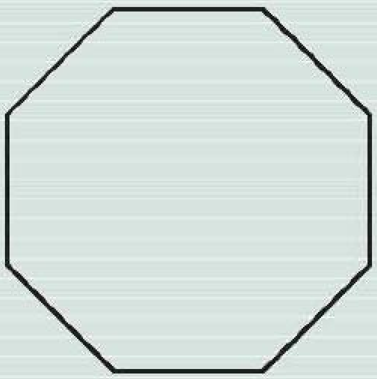
2. Write down the number of diagonals in the following boxes, and find out the pattern. Show Ans.

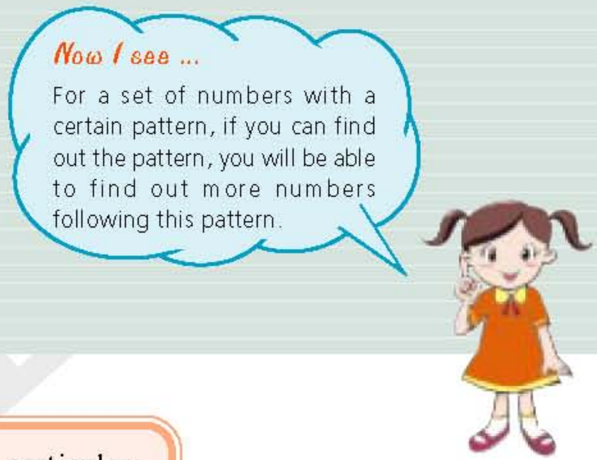


3. Observe the above pattern, then write down the number of diagonals of an octagon. Show Ans.

____

4. Draw all the diagonals in the following octagon to verify your answer in the above question. Show Ans.





A set of numbers arranged in a particular order forms a **sequence**. Each number in the sequence is called a **term** of the sequence.

e.g. In the sequence 2, 5, 9, 14, 20 obtained in Class Activity 2.3,

the 1st term = 2

the 2nd term = 5

the 3rd term = 9

Notes:

(a) There are usually many terms in a sequence. e.g. There are five terms in 2, 5, 9, 14, 20.

(b) The order of terms in a sequence is important. So, 2, 5, 9, 14, 20 and 9, 20, 14, 2, 5 are different sequences.

(c) In this chapter, we will only deal with sequences with simple patterns.

sequence 數列 term 項

Additional Example 2.9

Example 2.9 Writing down the terms of sequences

Observe the patterns of the following sequences, then write down the 5th term of each sequence.

(a) 4, 7, 10, 13, ______

(b) 6, 18, 54, 162, ______

Solution

(a) The pattern of the sequence:

$$4 \xrightarrow{+3} 7 \xrightarrow{+3} 10 \xrightarrow{+3} 13 \xrightarrow{+3} \underline{\qquad}$$

$\therefore$ The 5th term $= 13 + 3$

$= \underline{\underline{16}}$

(b) The pattern of the sequence:

$$6 \xrightarrow{\times 3} 18 \xrightarrow{\times 3} 54 \xrightarrow{\times 3} 162 \xrightarrow{\times 3} \underline{\qquad}$$

$\therefore$ The 5th term $= 162 \times 3$

$= \underline{\underline{486}}$

Alternative method:

(a) $4 = 1 + 3$

$7 = 1 + 3 + 3$

$10 = 1 + 3 + 3 + 3$

$13 = 1 + 3 + 3 + 3 + 3$

$\therefore$ The 5th term $= 1 + 3 + 3 + 3 + 3 + 3$

$= \underline{\underline{16}}$

(b) $6 = 2 \times 3$

$18 = 2 \times 3 \times 3$

$54 = 2 \times 3 \times 3 \times 3$

$162 = 2 \times 3 \times 3 \times 3 \times 3$

$\therefore$ The 5th term $= 2 \times 3 \times 3 \times 3 \times 3 \times 3$

$= \underline{\underline{486}}$

Note: From the above example, we can see that there may be more than one feasible pattern in each sequence.

Classwork 2.9

Solution

Observe the patterns of the following sequences, then write down the 5th term of each sequence.

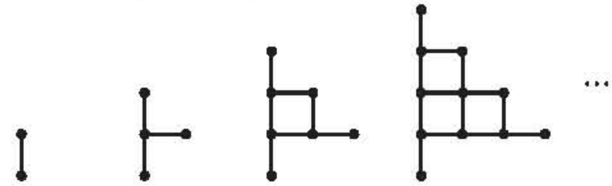
(a) 38, 31, 24, 17, ______

(b) 1 024, 512, 256, 128, ______

Additional Example 2.10

Example 2.10 Investigating the pattern of figures

The following is a sequence of figures.



The 1st figure The 2nd figure The 3rd figure The 4th figure

(a) According to the number of dots (•) and number of lines (—) in each figure, write down a sequence formed by the number of dots, and another sequence formed by the number of lines, where each sequence has four terms.

(b) Write down the 5th term of each sequence obtained in **(a)**.

(c) Draw the 5th figure. Do the number of dots and number of lines in the figure match the result of **(b)**?

Solution

(a) The sequence formed by the number of dots: 2, 4, 7, 11, ⋯

The sequence formed by the number of lines: 1, 3, 7, 13, ⋯

(b) [Analysis: 2, 4, 7, 11, ______
+2 +3 +4 +5]

The 5th term $= 11 + 5$

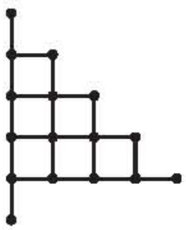
$= \underline{\underline{16}}$

[Analysis: 1, 3, 7, 13, ______
+2 +4 +6 +8]

The 5th term $= 13 + 8$

$= \underline{\underline{21}}$

(c) The 5th figure is as follows:

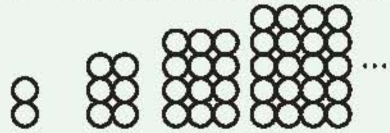


There are 16 dots and 21 lines, which match the result of **(b)**.

Classwork 2.10

Solution

The following is a sequence of figures.



The 1st figure The 2nd figure The 3rd figure The 4th figure

(a) According to the number of circles (○) in each figure, write down a sequence with four terms.

(b) Write down the 5th term of the sequence obtained in (a).

(c) Draw the 5th figure. Does the number of circles in the figure match the result of (b)?

B Some common sequences

The following are some common sequences and their patterns.

I. Square sequence

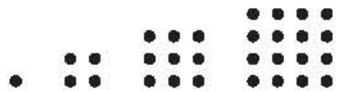
Figures with square pattern					…
Sequence	1,	4,	9,	16,	…
Pattern	1,	4,	9,	16,	…
		+3	+5	+7	
	or 1^2,	2^2,	3^2,	4^2,	…

Table 2.2

Note: The numbers 1, 4, 9, 16 in the **square sequence** are called **square numbers**.

II. Triangular sequence

Figures with triangular pattern					…
Sequence	1,	3,	6,	10,	…
Pattern	1,	3,	6,	10,	…
		+2	+3	+4	

Table 2.3

Note: The numbers 1, 3, 6, 10 in the **triangular sequence** are called **triangular numbers**.

Extension 2.1 Solution

The following are the first four terms of the square sequence and the triangular sequence.

Square sequence: 1, 4, 9, 16

Triangular sequence: 1, 3, 6, 10

Find the 5th term to the 8th term of each sequence.

square sequence 正方形數列　　square number 正方形數
triangular sequence 三角形數列　　triangular number 三角形數

III. Arithmetic sequence

Observe the patterns of the following sequences, can you find anything in common among them?

(a) 1, 3, 5, 7, 9, 11
+2 +2 +2 +2 +2

(b) 3, 6, 9, 12, 15, 18
+3 +3 +3 +3 +3

(c) 99, 98, 97, 96, 95, 94
−1 −1 −1 −1 −1

(d) 48, 44, 40, 36, 32, 28
−4 −4 −4 −4 −4

> An **arithmetic sequence** is a sequence in which the differences of any terms (except the first term) minus their preceding ones are the same.

I.T. Tool

This pattern is found in all of the sequences in **(a)** - **(d)**. Therefore they are all arithmetic sequences.

IV. Geometric sequence

Observe the patterns of the following sequences, can you find anything in common among them?

(a) 2, 4, 8, 16, 32, 64
×2 ×2 ×2 ×2 ×2

(b) 1, 0.1, 0.01, 0.001, 0.000 1, 0.000 01
$\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$

(c) 1, −3, 9, −27, 81, −243
×(−3) ×(−3) ×(−3) ×(−3) ×(−3)

> A **geometric sequence** is a sequence in which the quotients of dividing any terms (except the first term) by their preceding ones are the same.

I.T. Tool

This pattern is found in all of the sequences in **(a)** - **(c)**. Therefore they are all geometric sequences.

arithmetic sequence 等差數列 geometric sequence 等比數列

Additional Example 2.11

Example 2.11 Distinguishing between arithmetic sequences and geometric sequences

Observe the pattern of each of the following sequences, write down the next two terms of the sequence and determine what kind of sequence it is.

(a) 1, 5, 9, 13, 17, ⋯

(b) 3, 6, 12, 24, 48, ⋯

Classwork 2.11

Solution

Observe the pattern of each of the following sequences, write down the next two terms of the sequence and determine what kind of sequence it is.

(a) 25, 22, 19, 16, ⋯

(b) 2, −4, 8, −16, ⋯

Solution

(a) The pattern of the sequence:

1, 5, 9, 13, 17, ______, ______

+4 +4 +4 +4 +4 +4

$$\begin{aligned}\text{The 6th term} &= 17+4\\ &= \underline{\underline{21}}\end{aligned}$$

$$\begin{aligned}\text{The 7th term} &= 21+4\\ &= \underline{\underline{25}}\end{aligned}$$

It is $\underline{\underline{\text{an arithmetic sequence}}}$.

(b) The pattern of the sequence:

3, 6, 12, 24, 48, ______, ______

×2 ×2 ×2 ×2 ×2 ×2

$$\begin{aligned}\text{The 6th term} &= 48\times 2\\ &= \underline{\underline{96}}\end{aligned}$$

$$\begin{aligned}\text{The 7th term} &= 96\times 2\\ &= \underline{\underline{192}}\end{aligned}$$

It is $\underline{\underline{\text{a geometric sequence}}}$.

V. Fibonacci sequence

Observe the pattern of the following sequence:

1, 1, 2, 3, 5, 8, 13, ⋯

+ + + + +

The 1st term and the 2nd term are both 1, and starting from the 3rd term, each term is the sum of the previous two terms.

Fibonacci (1170 - 1250), an Italian mathematician, raised the question 'How many pairs of rabbits can be *breed* from one pair in a year?' in his work *Liber abaci*, which gave rise to the famous Fibonacci sequence.

Student's CD

I.T. Tool

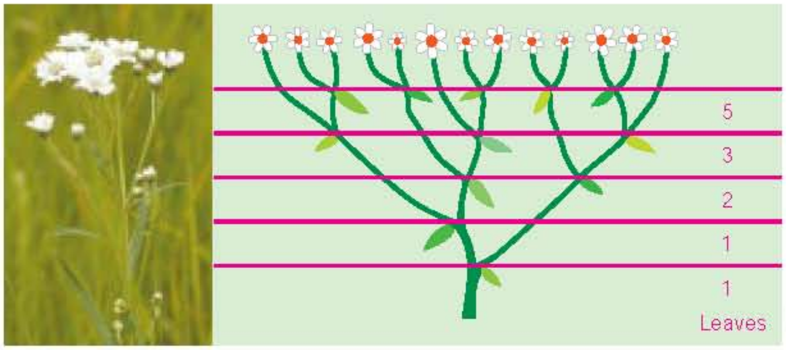
breed 繁殖 Liber abaci 算盤書

e.g. The 6th term = The 4th term + The 5th term

$= 3 + 5$

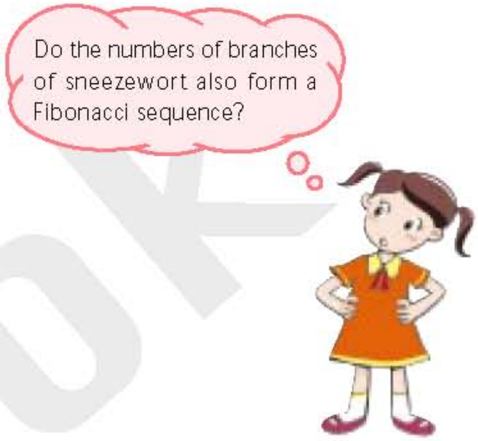
$= 8$

The sequence 1, 1, 2, 3, 5, 8, 13, … is called **Fibonacci sequence** and the numbers 1, 2, 3, 5, 8, 13 etc. in the sequence are called **Fibonacci numbers**. It is not difficult to find out that the growth of many natural creatures are connected with this sequence. (See Figure 2.1)



The numbers of leaves of *sneezewort* form a Fibonacci sequence.

Figure 2.1



Additional Example 2.12

Example 2.12 Writing down the terms of the Fibonacci sequence

It is known that the 6th term and the 7th term of the Fibonacci sequence are 8 and 13 respectively, find the 8th term and the 9th term of the sequence.

Solution

[Analysis:

…, 8, 13, ________ , ________]

The 8th term = The 6th term + The 7th term

$= 8 + 13$

$= \underline{\underline{21}}$

The 9th term = The 7th term + The 8th term

$= 13 + 21$

$= \underline{\underline{34}}$

Classwork 2.12

Solution

It is known that the 11th term and the 12th term of the Fibonacci sequence are 89 and 144 respectively, find the 13th term and the 14th term of the sequence.

Fibonacci sequence 斐波那契數列　Fibonacci number 斐波那契數　sneezewort 噴嚏麥

Skills Upgrading Corner 2.3

Solution

1. Observe the patterns of the following sequences, then write down the 6th term and the 7th term of each sequence.

(a) 92, 89, 86, 83, 80, ________, ________

(b) 6 561, 2 187, 729, 243, 81, ________, ________

Solution

2. It is known that the 16th term and the 18th term of the Fibonacci sequence are 987 and 2 584 respectively, find the 17th term and the 19th term of the sequence.

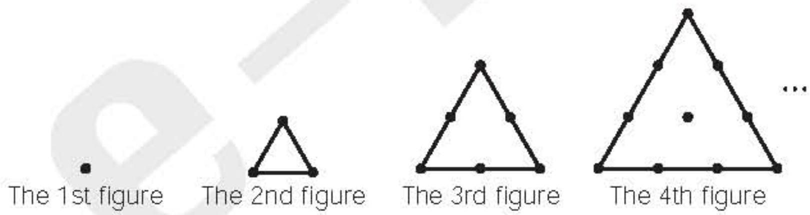
Exercise 2C

Level 1

1. For each of the following sequences of figures, draw the 5th figure.

(a)

(b)



2. Observe the patterns of the following sequences, then write down the 5th term and the 6th term of each sequence.

(a) 2, 7, 12, 17, ⋯ **(b)** 4, 12, 36, 108, ⋯

3. From 1 to 20 inclusive, write down all the

(a) triangular numbers. **(b)** square numbers.

4. Observe the pattern of each of the following sequences, write down the next three terms of the sequence and determine what kind of sequence it is.

(a) 2, 4, 6, 8, ⋯ **(b)** 5, 3, 1, −1, ⋯

(c) 1, 2, 4, 8, ⋯ **(d)** 384, 192, 96, 48, ⋯

Level 2

5. Each of the following pairs of numbers are the 7th term and the 8th term of an arithmetic sequence, find the 6th term of each sequence.
   (a) 17, 19
   (b) 28, 25

6. Each of the following pairs of numbers are the 4th term and the 5th term of a geometric sequence, find the 3rd term of each sequence.
   (a) 24, 48
   (b) 80, 40

7. It is known that the 9th term and the 10th term of the Fibonacci sequence are 34 and 55 respectively, find the 8th term and the 11th term of the sequence.

8. **(a)** It is given that 1 035 and 1 081 are two consecutive terms of the triangular sequence. Write down the next three terms.
   (b) Among the five terms in **(a)**, one of them is also a square number. Write down this number. If this number is the nth term of the square sequence, what is the value of n?

9. The following is a sequence of figures.

   (a) According to the number of lines (—) in each figure, write down a sequence with four terms.
   (b) Write down the 5th term of the sequence obtained in **(a)**.
   (c) Draw the 5th figure. Does the number of lines in the figure match the result of **(b)**?

2.4 The nth Term of a Sequence

Consider the sequence 2, 4, 6, 8, 10, $\cdots$, clearly, the nth term of the sequence can be represented by $2n$.

If the nth term of a sequence can be represented by an algebraic expression, we can use it to find each term of the sequence.

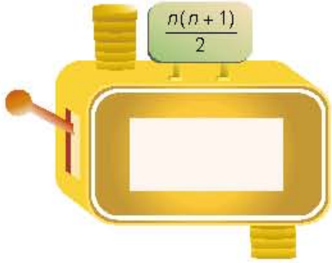
e.g. In the above sequence,

$$\begin{aligned}\text{the 28th term} &= 2(28)\\ &= 56\end{aligned}$$

Extension 2.2

Solution

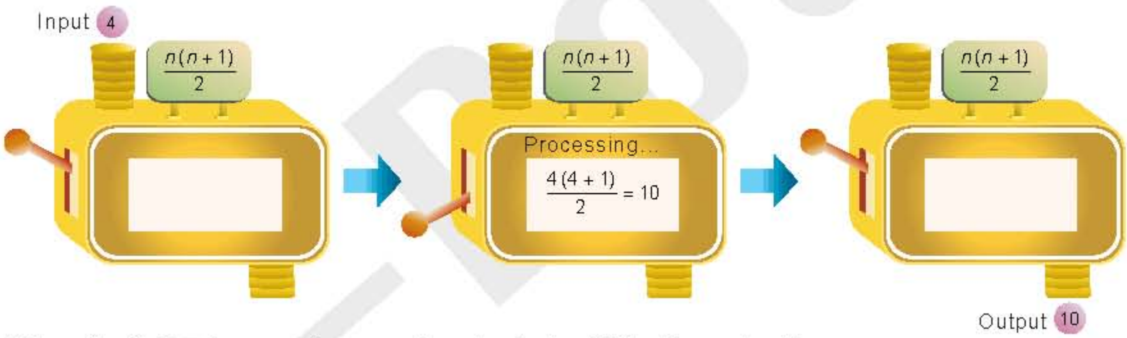
The following figure shows a number machine with an algebraic expression $\dfrac{n(n+1)}{2}$ to work out the nth term of the triangular sequence.



I.T. Tool

When a number is input, the number machine will substitute the number into n in $\dfrac{n(n+1)}{2}$, and then produce an output.

e.g. When 4 is input, the machine will produce an output of 10, which means that the 4th term of the triangular sequence is 10.



(a) When the following numbers are input, what will be the output?

Input	3	7	9	10
Output				

(b) If the same number is input each time, will the machine produce different output?

In fact, a value substituting into an algebraic expression is treated as an 'input', the operation is treated as a 'process', and the result obtained is treated as an 'output'.

Input ➡ *Process* ➡ *Output*

For an algebraic expression which represents the nth term of a sequence, when a value of n is substituted (input), the value of the nth term (output) can be obtained after the operation. Since the value of n can vary, n is called a **variable**.

variable 變數

Additional Example 2.13

Example 2.13 Finding the terms of a sequence

It is given that the nth term of a sequence is $5n+1$. Find the first four terms of the sequence.

Solution

The 1st term $= 5(1)+1$
$= \underline{\underline{6}}$

The 2nd term $= 5(2)+1$
$= \underline{\underline{11}}$

The 3rd term $= 5(3)+1$
$= \underline{\underline{16}}$

The 4th term $= 5(4)+1$
$= \underline{\underline{21}}$

Classwork 2.13

Solution

It is given that the nth term of a sequence is $100-7n$. Find the first four terms of the sequence.

Additional Example 2.14

Example 2.14 Finding the nth term of a sequence

A sequence 5, 10, 15, 20, $\cdots$ is given.

(a) Find an algebraic expression to represent the nth term.

(b) Using the result of **(a)**, find the 9th term of the sequence.

Solution

(a) The 1st term $= 5 = 5\times 1$
The 2nd term $= 10 = 5\times 2$
The 3rd term $= 15 = 5\times 3$
The 4th term $= 20 = 5\times 4$

According to the above pattern,
the nth term $= 5\times n$
$= \underline{\underline{5n}}$

(b) The 9th term $= 5\times 9$
$= \underline{\underline{45}}$

Note: Advanced mathematical knowledge is generally required for finding the nth term of a sequence based on some given terms. Thus this will be discussed in higher forms, and only simple examples are discussed in this chapter.

Classwork 2.14

Solution

A sequence 4, 7, 10, 13, $\cdots$ is given, where
the 1st term $= 4 = 1+3\times 1$,
the 2nd term $= 7 = 1+3\times 2$,
the 3rd term $= 10 = 1+3\times 3$,
the 4th term $= 13 = 1+3\times 4$.

(a) According to the above expressions, find an algebraic expression to represent the nth term.

(b) Using the result of (a), find the 20th term of the sequence.

Skills Upgrading Corner 2.4

Solution

1. It is given that the nth term of a sequence is $n(n + 1)$. Find the first four terms of the sequence.

Solution

2. A sequence 5, 9, 13, 17, $\cdots$ is given, where

the 1st term $= 5 = 1 + 4 \times 1$,

the 2nd term $= 9 = 1 + 4 \times 2$,

the 3rd term $= 13 = 1 + 4 \times 3$,

the 4th term $= 17 = 1 + 4 \times 4$.

(a) According to the above expressions, find an algebraic expression to represent the nth term of the sequence.

(b) Using the result of **(a)**, find the 8th term.

Exercise 2D

Level 1

1. Each of the following algebraic expressions represents the nth term of a sequence, find the first three terms of each sequence.

(a) $4n + 3$

(b) $10 - 2n$

(c) $n(n + 2)$

(d) $\dfrac{n}{n+1}$

2. A sequence 3, 6, 9, 12, $\cdots$ is given.

(a) Write down the 5th term of the sequence.

(b) Find an algebraic expression to represent the nth term of the sequence.

Level 2

3. A sequence $\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}, \dfrac{1}{5}, \cdots$ is given.

(a) Write down the 5th term of the sequence.

(b) Find an algebraic expression to represent the nth term of the sequence.

(c) Using the result of **(b)**, find the 10th term of the sequence.

4. A sequence 6, 7, 8, 9, ⋯ is given.

(a) Write down the 6th term of the sequence.

(b) Find an algebraic expression to represent the nth term of the sequence.

(c) Using the result of **(b)**, find the 9th term of the sequence.

5. A sequence 2, 5, 8, 11, ⋯ is given, where

the 1st term $= 2 = 1\times 3-1$,

the 2nd term $= 5 = 2\times 3-1$,

the 3rd term $= 8 = 3\times 3-1$,

the 4th term $= 11 = 4\times 3-1$.

(a) According to the above expressions, find an algebraic expression to represent the nth term of the sequence.

(b) Using the result of **(a)**, find the 8th term of the sequence.

6. The following is a sequence of figures.

The 1st figure The 2nd figure The 3rd figure

(a) Draw the next two figures.

(b) Complete the table below.

Order of figures	Number of dots
1	
2	
3	
4	
5	

(c) Find the number of dots in the nth figure.

(d) Find the number of dots in the 15th figure.

Chapter Summary

Chapter Summary

A. Term Introduced

[This is a quiz to check your understanding of some special terms in this chapter. Match items in column A to column B appropriately.]

	Column A				Column B
1.	Algebra	•	•	**(a)**	A set of numbers arranged in a particular order.
2.	Algebraic expression	•	•	**(b)**	An equality representing the relations between numbers.
3.	Formula	•	•	**(c)**	The method of replacing letters with values of the numbers.
4.	Substitution	•	•	**(d)**	The knowledge of using letters to represent numbers for the investigation of the relations between numbers and the rules of arithmetic.
5.	Sequence	•	•	**(e)**	A number in a sequence.
6.	Term	•	•	**(f)**	The expression obtained by addition, subtraction, multiplication or division of numbers and letters representing numbers.

B. Fact to Remember

1. Square sequence

1, 4, 9, 16, …

or 1^2, 2^2, 3^2, 4^2, …

2. Triangular sequence

1, 3, 6, 10, …

3. Arithmetic sequence

An arithmetic sequence is a sequence in which the differences of any terms (except the first term) minus their preceding ones are the same.

e.g. 1, 4, 7, 10, 13, 16

4. Geometric sequence

A geometric sequence is a sequence in which the quotients of dividing any terms (except the first term) by their preceding ones are the same.

e.g. 2, 6, 18, 54, 162, 486

5. Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, $\cdots$

Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

Section

1. **(a)** Karen is now x years old and Walter is 4 times as old as she is. 2.1B

After five years, Walter will be __________ years old.

(b) Express the following statement by using an algebraic expression.

Subtract p from the product of q and 2, and then divide the difference by 3 times r.

2. Simplify the following expressions. 2.1B

(a) $4n-2n+3n$

(b) $12+4x-7+8x$

3. **(a)** The formula for the area A cm^2 of a rectangle is $A=\ell b$, where ℓ cm and b cm are the length and breadth respectively. If $\ell=3$ and $b=4$, then $A=$ __________. 2.2

(b) If $x=2$, $y=3$ and $z=1$, find the value of $\dfrac{xy+10}{z^2}$.

4. **(a)** The following is a sequence of figures: 2.3A

The 1st figure   The 2nd figure   The 3rd figure

There are ________ lines in the 4th figure.

(b) Observe the pattern of the following sequence, then write down the 5th term and the 6th term of the sequence.

1, 3, 9, 27, __________ , __________

5. Observe the pattern of each of the following sequences, write down the next two terms of the sequence and determine what kind of sequence it is. 2.3B

(a) $-1, 3, 7, 11, \cdots$

(b) $0.1, 1, 10, 100, \cdots$

6. **(a)** It is given that the nth term of a sequence can be represented by $2n+3$. The 3rd term of this sequence is __________ . 2.4

(b) Given a sequence 6, 12, 18, 24, $\cdots$, find an algebraic expression to represent the nth term of the sequence.

Revision Exercise 2

Level 1

1. Express the following statements by using algebraic expressions.

(a) Subtract the sum of 3 and $2x$ from the product of y and z.

(b) Divide the sum of x and $2y$ by the sum of y and $2z$.

(c) The difference of p minus q is divided by 2 times r.

(d) Add $3c$ to the sum of a and the product of 2 and b.

2. Simplify the following expressions.

(a) $2m+4m-m$ **(b)** $6x+8-4+2x$

(c) $3a+4a+2b+6b$ **(d)** $7p-5q-4p+6q$

3. If $M = a(x + y)$, find the value of M for $a = 5$, $x = 3$ and $y = 2$.

4. If $I = \dfrac{PRN}{100}$, find the value of I for $P = 2\ 000$, $R = 8$ and $N = 3$.

5. If $f = \dfrac{uv}{u + v}$, find the value of f for $u = 6$ and $v = 3$.

6. If $y = ax^3$, find the value of y for $a = 5$ and $x = 2$.

Use the simplest algebraic expressions to answer the following questions. (7 – 9)

7. A pen costs \$$a$, a pencil costs \$$b$, how much does it cost for c pens and c pencils?

8. The smallest number of three consecutive even numbers is $2m$, find their sum.

9. Peggy is m years old. Sammy is 6 years older than Peggy. Bill is 5 years younger than Peggy. Find the sum of their ages.

10. Observe the pattern of each of the following sequences, write down the next two terms of the sequence and determine what kind of sequence it is.

(a) 2, 11, 20, 29, 38, ⋯

(b) 24, 19, 14, 9, 4, ⋯

(c) 5, 10, 20, 40, 80, ⋯

(d) 81, –27, 9, –3, 1, ⋯

Level 2

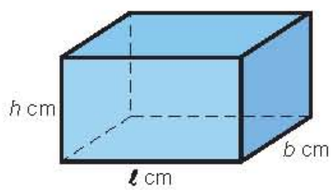
11. (a) A teacher evenly distributes a certain number of stickers to n children. Each child gets 5 stickers, and 3 are left. How many stickers does the teacher have at first?

(b) If there are 9 more stickers, all stickers will be distributed. How many stickers will each child get?

12. The figure shows a cuboid. It is known that the total surface area A cm^2 of a cuboid can be found by the formula $A = 2(\ell b + bh + \ell h)$, where ℓ cm is the length, b cm is the breadth and h cm is the height.

(a) If $\ell = 4$, $b = 3$ and $h = 2$, find the value of A.

(b) If $\ell = 2$, $b = 1.5$ and $h = 5$, find the value of A.



13. In year 2006, the *admission fees* of the Hong Kong Wetland Park for an adult and a child are \$30 and \$15 respectively. It is given that the total admission fee for x adults and y children is \$$E$.

(a) Write down a formula about E.

(b) If Mr. Chan and his wife bring 3 children to the Hong Kong Wetland Park, what is the total amount of their admission fees?

14. The number of diagonals D of a polygon can be obtained by the following formula:

$$D=\frac{n^2-3n}{2}$$

where n is the number of sides of the polygon.

(a) How many diagonals does a 10-sided polygon have?

(b) How many diagonals does a triangle have?

15. A sequence 2, 5, 10, 17, $\cdots$ is given, where

the 1st term $=2=1+1^2$,

the 2nd term $=5=1+2^2$,

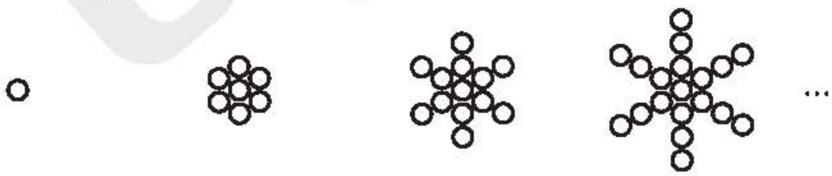
the 3rd term $=10=1+3^2$,

the 4th term $=17=1+4^2$.

(a) According to the above expressions, find an algebraic expression to represent the nth term of the sequence.

(b) Using the result of **(a)**, find the 9th term of the sequence.

16. The following is a sequence of figures.



The 1st figure   The 2nd figure   The 3rd figure   The 4th figure

(a) According to the number of circles (○) in each figure, write down a sequence with four terms.

(b) Write down the 5th term of the sequence obtained in **(a)**.

(c) Draw the 5th figure. Does the number of lines in the figure match the result of **(b)**?

admission fee 入場費

17. May uses toothpicks with the same length to form the following sequence of figures.

The 1st figure   The 2nd figure   The 3rd figure   …

(a) Draw the next two figures.

(b) Complete the table below.

Order of figures	1	2	3	4	5
Number of toothpicks					

(c) Find the number of toothpicks used in the nth figure.

(d) Find the number of toothpicks used in the 25th figure.

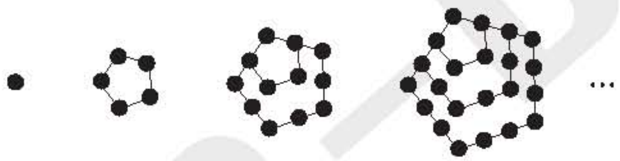
18. Sophia is now n years old. She is $\frac{1}{4}$ of the age of Tony but 6 years older than Phoebe.

(a) Find the present ages of Tony and Phoebe.

(b) After 2 years, what is the sum of the ages of the three of them?

(c) If the sum of their ages after 2 years is half of the age of Uncle Yeung, what is the present age of Uncle Yeung?

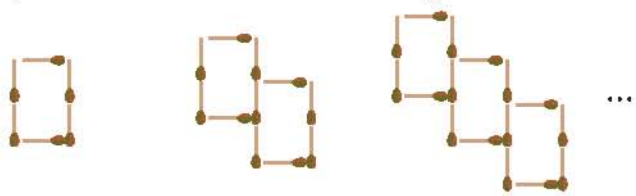
19. Observe the pattern of the following *pentagonal numbers*.



1   5   12   22   …

Write down all pentagonal numbers between 30 and 60.

20. Kary uses matches with the same length to form the following sequence of figures.



The 1st figure   The 2nd figure   The 3rd figure   …

(a) Draw the next two figures.

(b) Verify that the number of matches used in the nth figure is $5n+1$.

(c) If there are 80 matches, can they be used to form the 16th figure? Explain your answer.

pentagonal number 五邊形數

MC Question

21. Which of the following can represent the algebraic expression $(6x+6)y^2$?

A. The product of 6 and x is added to the product of 6 and the square of y.

B. The product of x and 6 is added to the product of y and 6.

C. 6 plus the product of 6 and x, and then multiply the sum by 2 times y.

D. 6 plus the product of x and 6, and then multiply the sum by the square of y. ☐

22. Simplify $a \times a \times b \times 4a \times 2b$.

A. $6ab$

B. $8ab$

C. $6a^3b^2$

D. $8a^3b^2$ ☐

23. The membership fee income $\$I$ of a basketball club can be obtained by the following formula:

$$I = 50n$$

where n is the number of members.

If there are 50 members in the basketball club, what is the membership fee income?

A. \$50

B. \$100

C. \$250

D. \$2 500 ☐

24. The following is a sequence of figures.

The 1st figure　The 2nd figure　The 3rd figure

How many lines are there in the 5th figure?

A. 18

B. 20

C. 22

D. 24 ☐

25. Which of the following are both triangular number and square number?

I. 1

II. 16

III. 36

A. I only

B. I and III only

C. II and III only

D. I, II and III ☐

26. The nth term of the sequence $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\cdots$ is

A. $\dfrac{2n}{n+2}$.

B. $\dfrac{n+1}{3n}$.

C. $\dfrac{n-1}{n+1}$.

D. $\dfrac{n+1}{n+2}$. ☐

Problem-solving and Exploring

Hint for the Title Page Question

In the following, E, F, I, N, O, R, S, T, X and Y each represents a number from 0 to 9 without repetition.

$$
\begin{array}{rccccc}
 & F & O & R & T & Y \\
 & & & T & E & N \\
+) & & & T & E & N \\
\hline
 & S & I & X & T & Y
\end{array}
$$

(a) If the units digit of Y + N + N is Y, find the possible values of N.

(b) Consider the results of T + E + E and **(a)**, find the values of N and E.

(c) If the value of O is 9, what is the value of I?

(d) What is the relation between F and S?

(e) Write down the number each letter represents.

Additional Question

1. Invite one of your classmates to do the following calculation.

Step 1: Ask your classmate to multiply his / her month of birth by 4 and then add 13 to the product.

Step 2: Multiply the result by 25 and then add his / her day of birth to the product.

Step 3: Subtract 200 from the result, then ask your classmate to tell you the result.

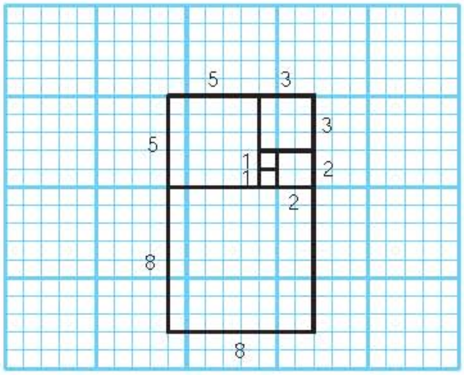
To find out your classmate's birthday, you only have to subtract the result in step 3 by 125. The day of birth can be read from the units and tens digits of the end result, and the month of birth can be read from the hundreds and thousands digits (if applicable). (e.g. If the end result is 803, your classmate's birthday is 3 August.)

(a) Let m be the month of birth, d be the day of birth, write down the formula for finding one's birthday through the calculation above.

(b) Substitute your month and day of birth into the above formula to verify it.

2. **(a)** Write down the next three terms for the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13.

(b) The following rectangle is formed by several squares, where the sides of these squares form a Fibonacci sequence. Using the result of **(a)**, continue to expand the following rectangle to the size of 34×55.



(c) Hence, find the value of $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2 + 21^2 + 34^2$.